## Precalculus

## **Course Description**

Precalculus is a program of mathematical studies focusing on the development of the student's ability to understand and apply the study of functions and advanced mathematics concepts to solve problems. The course will include an in-depth study of polynomial, rational, exponential, logarithmic, and trigonometric functions. Other topics studied are sequences, series, vectors, conic sections, parametric equations, and polar curves. Emphasis is placed on active participation through modeling, technology lab activities, group activities, and communication in mathematics.

Students are expected to use technology, including graphing calculators, computers, and data-gathering equipment throughout the course. *Graphing calculators should be an integral part of all instruction*.

## **Recommended Prerequisites**

It is recommended that a student successfully complete Algebra 2 before taking Precalculus. Students who need a stronger background could follow Algebra 2 with Algebra 3 and then Precalculus. Algebra 3 is not part of the defined program and will need to be taught as an innovative approach course.

Boldfaced items indicate additional material to be covered in Precalculus at the honors level. All topics should be taught in greater depth and difficulty at the honors level.

## **Course Outline**

- I. Functions.
  - A. Characteristics and representations of functions.
    - 1. Determine the domain and range from algebraic representations, graphs, and tables. **Precalculus: I.A.2., II.A.1., II.B.3.**
    - 2. Describe symmetry of even and odd functions.

      I.A.5.

      Precalculus:
  - B. Operations on functions.
    - 1. Apply basic transformations. **Precalculus: I.B.1.** 
      - a.  $a \bullet f(x), f(x) + d, f(x c), f(b \bullet x)$ .
      - b. |f(x)|, f(|x|).
    - 2. Perform operations and describe the procedures and results verbally, numerically, algebraically, and graphically. **Precalculus: I.B.2.** 
      - a. Composition and decomposition.
      - b. Inverses.
- II. Polynomial and rational functions.
  - A. Polynomial functions.

- 1. Describe the general shape of the graph and the effect of transformations on the domain and range. **Precalculus: I.A.1.,II.A.1.**
- 2. Recognize the connections among the significant points of a function, the graph of the function, and the algebraic representation of the function. Precalculus: I.A.6.
  - a. Roots.
  - b. Maximum points and minimum points.
- 3. Investigate continuity and end behavior. Precalculus: I.A.7.
- 4. Solve equations and inequalities using graphs, tables, algebraic methods, and technology. Precalculus: II.A.3.
- B. Rational functions.
  - 1. Describe the general shape of the graph and the effect of transformations on the domain and range. **Precalculus: I.A.1., II.A.1.**
  - 2. Investigate continuity, asymptotes, and limits. Precalculus: I.A.7., II.A.1.
  - 3. Solve equations and inequalities using graphs, tables, algebraic methods, and technology. Precalculus: II.A.3.
  - 4. Decompose a rational expression into partial fractions.
- C. Problem situations.
  - 1. Analyze a verbal, graphical, or tabular representation of a polynomial or rational function. Precalculus: II.A.2.
  - 2. Analyze a problem situation by formulating an equation or an inequality. Precalculus: II.A.4.
- Exponential and logarithmic functions. III.
  - A. Exponential functions.
    - 1. Describe the general shape of a graph and the effect of transformations on the domain and range. **Precalculus: I.A.1., II.B.2.**
    - 2. Investigate asymptotic behavior. Precalculus: II.B.2.
    - 3. Investigate exponential properties graphically and algebraically.
      - Precalculus: I.B.3.
    - 4. Solve exponential equations and inequalities using graphs, tables, algebraic methods, and technology, including reasonableness of solutions.

Precalculus: II.B.3.,4.

- B. Logarithmic functions.
  - 1. Describe the general shape of a graph and the effect of transformations on the domain and range. **Precalculus: I.A.1., II.B.2.**
  - 2. Develop the connection between exponential and logarithmic functions. Precalculus: II.B.1.
  - 3. Investigate asymptotic behavior. Precalculus: II.B.2.
  - 4. Investigate logarithmic properties graphically and algebraically.

Precalculus: I.B.3.

5. Solve logarithmic equations and inequalities using graphs, tables, algebraic methods, and technology, including reasonableness of solutions.

Precalculus: II.B.3., 4.

C. Problem situation.

- 1. Analyze a verbal, graphical, or tabular representation of an exponential or logarithmic function. **Precalculus: II.B.5.**
- 2. Analyze a problem situation by formulating an equation or inequality. **Precalculus: II.B.5.**
- 3. Solve rate of change problems, such as inflation, spread of disease, population growth, tax brackets, pollution, or other such problems.

  Precalculus: II.B.6.
- 4. Analyze graphical data gathered by technical equipment. **Precalculus:** II.B.7.
- IV. Trigonometric functions.
  - A. Circular functions.
    - 1. Use the wrapping function to define and evaluate all six trigonometric functions. **Precalculus: I.A.3.**
    - 2. Describe the general shape of a graph and the effect of transformations on the domain and range. **Precalculus: I.A.1.**,
    - 3. Investigate transformations of graphs including periodicity, amplitude, phase shift, and vertical shift.
    - 4. Investigate identities graphically, and verify algebraically. **Precalculus: I.B.3.**
    - 5. Solve trigonometric equations.
  - B. Triangle trigonometry.
    - 1. Evaluate all six trigonometric functions using a right triangle.

      Precalculus: I.A.3.
    - 2. Solve problems using the law of sines and law of cosines. **Precalculus:** II.C.3.
    - 3. Find the area of a triangle.
  - C. Vectors.
    - 1. Model situations defined by magnitude and direction. **Precalculus: III.C.1**, .2.
    - 2. Find distance graphically. **Precalculus: III.C.3.**
  - D. Problem situations.
    - 1. Analyze a verbal, graphical, or tabular representation of a trigonometric function. Precalculus: II.C.1., 2., 3.
    - 2. Analyze a problem situation by formulating an equation or inequality. **Precalculus: II.C.1., 2., 3.**
    - 3. Analyze graphical data gathered by technical equipment. **Precalculus:** II.C.1., 2., 3.
  - E. Inverse trigonometric functions.
    - 1. Describe the general shape of a graph and the effect of transformations on the domain and range.
    - 2. Evaluate inverse trigonometric functions.
- V. Conic sections, parametric representations, and polar representations.
  - A. Conic sections.
    - 1. Use conic sections to model motion and planetary motion. **Precalculus: III.B.1.**

- 2. Use conic sections to model the reflective properties of light and sound. **Precalculus: III.B.2.**
- 3. Solve systems of second-degree equations and inequalities.
- B. Parametric equations.
  - 1. Convert between rectangular and parametric form. **Precalculus: III.B.3.**
  - 2. Graph parametric equations. **Precalculus: III.B.3.**
  - 3. Use parametric equations to simulate problems involving motion. **Precalculus: III.B.4.**
- C. Polar equations.
  - 1. Convert between rectangular and polar form. Precalculus: III.B.5.
  - 2. Graph polar equations. **Precalculus: III.B.5.**
  - 3. Write complex numbers in polar form.
  - 4. Find products, quotients, powers, and roots of complex numbers in polar form.
- VI. Sequences and series.
  - A. Sequences.
    - 1. Represent patterns using arithmetic and geometric sequences. **Precalculus: III.A.1**.
    - 2. Solve real-life problems using arithmetic, geometric, and other sequences. **Precalculus: III.A.2.**
    - 3. Describe limits of sequences. **Precalculus: III.A.4.**
  - B. Series.
    - 1. Represent patterns using arithmetic and geometric series, including sigma notation
    - 2. Solve real-life problems using arithmetic, geometric, and other series.
    - 3. Investigate convergent and divergent series. **Precalculus: III.A.4.**
  - C. Problem solving.
    - 1. Solve problems including sums, binomial expansion, the binomial theorem, combinations, and Pascal's triangle. **Precalculus: III.A.5.**
    - 2. Use the principle of mathematical induction. **Precalculus: III.A.6.**
    - 3. Apply informal concepts of successive approximations, upper and lower bounds, and limits in measurement situations, such as estimated lengths of curves, areas of curved regions, and volume of curved solids.

Precalculus: III.A.3.

- VII. Iteration and fractals.
  - A. Iteration functions using real numbers.
  - **B.** Investigate fractals.